Enhancing aperiodic stochastic resonance through noise modulation

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We show that the conventional stochastic resonance (SR) effect for aperiodic signals in a model neuron can be enhanced by modulating the intensity of the input noise (which could be introduced artificially in bioengineering applications) with either the input signal or the unit's output rate signal. We analyze SR enhancement theoretically and numerically. We discuss how this work provides the theoretical foundation for the development of an optimal noise-based technique for enhancing sensory function. © *1998 American Institute of Physics.* [S1054-1500(98)00303-6]

Stochastic resonance (SR) is a phenomenon where the response of a nonlinear system to a weak signal is enhanced by the presence of noise. SR has been observed in a broad range of different systems. Here we focus on SR in excitable systems, such as neurons. It has been shown that noise can serve to enhance the response of a sensory neuron to a subthreshold input signal. This points to potential bioengineering applications where noise can be artificially applied to improve sensory function. Given the possible practical benefits of SR, it is of interest to devise methods that could enhance the SR effect. Here we show that such an enhancement can be achieved in a neuronal model by modulating the intensity of the input noise with either the input signal or the unit's output rate signal.

I. INTRODUCTION

Stochastic resonance (SR)[1] is a phenomenon where the response of a nonlinear system to a weak signal is enhanced by the presence of noise. SR has been examined theoretically and experimentally in a wide variety of physical[2] and biological[3,4] systems. SR was originally proposed for systems with periodic input signals,[2] but recently, it was shown that SR-type dynamics can be characterized in systems with aperiodic inputs.[5–9] This general type of behavior was termed aperiodic stochastic resonance (ASR).[10] (Recently, Neiman *et al.*[11] showed, using linear response theory, that ASR is equivalent to conventional SR.) These developments together have pointed to the possible beneficial effects of noise on the dynamics of nonlinear systems, e.g., these results have suggested that SR-type dynamics could be exploited in the design of signal-detection devices and techniques.Given the ubiquity and possible practical utility of SR-type dynamics, it is of interest to explore how the response of a nonlinear system to an arbitrary weak signal could be maximally enhanced. Several groups have considered enhancing SR through the use of uncoupled[12] and coupled[13] networks of nonlinear elements. Previously, we showed that a simple summing network of independent nonlinear units could enhance and broaden the range of SR and ASR.[6] Here we focus on enhancing ASR (and hence SR) in a single nonlinear unit. Specifically, we demonstrate that the SR effect in a single excitable unit, such as a sensory neuron, can be enhanced by modulating the input noise intensity with the input signal or the unit's output rate signal through feedback. This effect exploits a cooperative effect between additive SR and multiplicative SR.[14–16]

The use of feedback in this way might be used in biological systems. For instance, it has been found that the response variance of some neurons in the cat's visual cortex increases with the mean response amplitude.[17] Another motivation for this problem comes from bioengineering. On the basis of earlier studies,[4,5,7,8] it has been suggested that noise-based techniques could be used to improve the function of neurophysiological sensory systems (e.g., the touch-sensation system and the proprioceptive system). With such techniques, noise would be introduced artificially into sensory neurons in order to improve their abilities to detect arbitrary subthreshold signals. Noise would be the means of choice for such purposes since most sensory neurons adapt away DC inputs but do not adapt away the effects of noise. It is possible with bioengineering sensors to measure the input stimulus to a sensory neuron (e.g., the change in joint angle in the case of a proprioceptor), as well as the output rate signal from a sensory neuron.

Here we show how such information can be utilized to optimize the response of a sensory neuron to arbitrary subthreshold stimuli. We provide the theoretical foundation for the development of an optimal noise-based technique for enhancing sensory function.

II. CORRELATION MEASURES

We consider two cross-correlation measures for characterizing SR-type dynamics:[5,8]

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$$C_0 = \langle \overline{S(t)R(t+\tau)} \rangle, \tag{2.1}$$

$$C_1 = \left\langle \frac{\overline{S(t)R(t+\tau)}}{[\overline{S^2(t)}]^{1/2}[\overline{(R(t)-\overline{R(t)})^2}]^{1/2}} \right\rangle, \tag{2.2}$$

where $S(t)$ is the zero-mean input signal, $R(t)$ is the response of the system characterized by the mean transition rate, the overbar denotes an average over time, and the angled brackets denote ensemble averages over the noise distribution. Generally, $R(t)$ is formed by lowpass filtering the output pulse train of the neuron. These measures are defined as the maximum of their respective correlation functions for a fixed time lag τ, which allows for inherent delays in the system. Both C_0 and C_1 are scalar measures of the coherence between the input and the output. They are applicable for all types of input signals, periodic and aperiodic. For the specific case of a periodic input, the measure C_0 corresponds to the transferred signal strength and C_1 corresponds to the output signal-to-noise ratio (SNR), which has been the traditional SR measure.[1]

The correspondence between C_1 and the output SNR can be seen if we consider a periodic input signal $S(t) = \cos(\omega_0 t + \phi)$, where ω_0 is an arbitrary frequency and ϕ is some arbitrary phase. Consider the numerator of C_1 given by $\overline{SR} = T^{-1}\int_0^T \cos(\omega t + \phi)R(t)dt \equiv \hat{s}(\omega)$, where T is an averaging window. If we compute $\hat{s}(\omega)$ for finite T, it will be a random variable that must be ensemble averaged. However, in the limit of infinite T, the variance goes to zero and $\hat{s}(\omega)$ becomes a constant that captures the periodic component of $R(t)$, while $\hat{s}^2(\omega)$ represents the normalized power of $R(t)$ with frequency ω and phase ϕ. [For a periodic signal $S(t)$, the time average $\overline{SR}$ over infinite time is equivalent to $\langle \overline{SR} \rangle$, the ensemble average of the time average over one period.] Similarly, in this limit, the denominator factor $\overline{(R(t)-\overline{R(t)})^2} \equiv \hat{r}^2$ corresponds to the normalized total power contained in the zero-mean output signal. This is also the total power in frequency space and we can express it as $\hat{r}^2 = \hat{s}^2 + \hat{n}^2$, where $\hat{s}^2$ represents the output power at the signal frequency (and phase) and $\hat{n}^2$ represents the noise power. This leads to the relation $C_1^2 \propto \mathrm{SNR}/(1+\mathrm{SNR})$, where $\mathrm{SNR} = \hat{s}^2/\hat{n}^2$. Thus, C_1 can be thought of as a ''phasic'' output SNR, where both frequency and phase information is used.

For aperiodic input signals, the lack of a prescribed frequency makes C_0 and C_1 natural measures for characterizing SR-type dynamics (i.e., ASR). In addition, C_1 is related to the information transfer rate — for small signal amplitudes, it is directly proportional to the square root of the Shannon transinformation.[9] Here we investigate how these measures can be optimized. In particular, we concentrate on enhancing the maximum values of C_0 and C_1 as a function of the noise intensity.

III. ASR ENHANCEMENT

Our method for ASR enhancement is applicable to any generic excitable system that has two stable states. The system is modulated by a subthreshold input signal which by itself cannot induce the system to change states. However, with the addition of thermal noise, the system can be induced to change states. This change of state, which we call a firing event, could be the escape of a particle from a potential well or the generation of an action potential by a neuron. The firing rate of the system will be a function of both the noise intensity and the input signal.

For concreteness, we demonstrate ASR enhancement for the FitzHugh-Nagumo (FHN) neuronal model, which we write in the form[5]

$$\epsilon \dot{v} = -v(v^2 - \tfrac{1}{4}) - w + A_T - \gamma(t) + \xi(t), \tag{3.1}$$

$$\dot{w} = v - w, \tag{3.2}$$

where ϵ is a small parameter, $v(t)$ is a voltage variable, $w(t)$ is a recovery variable, $A_T = -5/(12\sqrt{3})$ is a threshold voltage, $\gamma(t)$ is the input signal, and $\xi(t)$ is a white, zero-mean Gaussian noise term with an autocorrelation function $\langle \xi(t)\xi(s) \rangle = 2D\delta(t-s)$, where D is the noise intensity and the angular brackets $\langle \cdot \rangle$ denote ensemble averaging over the distribution of ξ. We express the input signal as $\gamma(t) = B - S(t)$, where B is the DC part corresponding to the signal-to-threshold distance, and $S(t)$ is the zero-mean fluctuating part. Below we show that modulating D with either $S(t)$ or the output rate can enhance the SR effect in the FHN model. Our results can also be generalized to other nonlinear systems.

Previously, we showed that the FHN model (and other neuronal models) for a subthreshold signal could be approximated by a one-dimensional barrier-escape problem.[5,8] The ensemble-averaged firing rate had the form of the Kramers escape rate:[5,8,18]

$$\langle R(t) \rangle \simeq Q \exp\left[\frac{-\theta}{D} + \frac{\Delta}{D} S(t)\right], \tag{3.3}$$

where $S(t)$ is the zero-mean input signal, D is the noise intensity, $Q = B/2\sqrt{3}\epsilon$ is a factor which sets the rate scale, $\theta = 2\sqrt{3}\epsilon B^3/3$ is the signal-to-threshold distance, and $\Delta = 2\epsilon B^2$ is the signal sensitivity or gain. In Ref. 8, we showed that a wide variety of models have escape rates with the general form of Eq. (3.3).

We make the ansatz that the unaveraged rate takes the form[5,8]

$$R(t) = \langle R(t) \rangle + \eta(t), \tag{3.4}$$

where η is a stochastic variable satisfying $\overline{\eta(t)} = 0$ and $\overline{\eta^2(t)} = \sigma(D)$. For low firing rates, we can use the approximation that the firing is a Poisson process so $\sigma(D) = K_1 Q \exp(-\theta/D)$, and K_1 is a constant proportional to the inverse of the window length used to obtain $R(t)$.[8,19]

Using the rate expression Eq. (3.3) in the SR measures (2.1) and (2.2), we obtain[5,8]

$$C_0 \simeq Q\Delta D^{-1} \exp[-\theta/D]\overline{S^2(t)}, \tag{3.5}$$

$$C_1 \simeq \left[1 + \frac{K_1 D^2 \exp(\theta/D)}{\Delta^2 Q \overline{S^2(t)}}\right]^{-1/2}. \tag{3.6}$$

Here we have assumed that the signal amplitude is small compared to the noise amplitude and $S(t)$ is slowly varying compared to the slowest time scale of the system. It is important to note that these approximations are taken only to facilitate the theoretical estimates and are not essential for the phenomenon. Maximizing Eqs. (3.5) and (3.6) with respect to D gives maximum values of

$$\max(C_0) \simeq Q\Delta\theta^{-1}\exp(-1)\overline{S^2(t)}, \tag{3.7}$$

$$\max(C_1) \simeq \left[1 + \frac{K_1\theta^2\exp(2)}{4\Delta^2 Q\overline{S^2(t)}}\right]^{-1/2}. \tag{3.8}$$

We now investigate ways to maximize Eqs. (3.7) and (3.8). We find that the maximum value of C_0 and C_1 can be enhanced by either decreasing the threshold distance θ, increasing the firing-rate factor Q, or increasing the signal sensitivity Δ. The most direct way is to modify the input signal. The effective signal-to-threshold distance θ could be decreased by adding a DC contribution to the input signal. In fact, if enough DC were added, the signal could be pushed above threshold. (Jung considered the effects of noise on suprathreshold signals.[20]) However, it is known that many types of sensory neurons adapt away the effects of DC inputs.[21] For adapting neurons, the addition of DC to an input signal will not enhance the output SNR. The neuron will simply readjust its threshold.

Here we focus on enhancing SR dynamically by modulating the input noise intensity with the input signal or with the output rate through feedback. Neurons generally do not adapt away the effects of noise. This has the effect of increasing the signal sensitivity Δ. It has been shown that systems can exhibit SR-type dynamics if the signal modulates the noise multiplicatively.[14–16] Here we show that this effect works cooperatively with conventional additive SR. We also generalize the effect to aperiodic signals and output rate-modulated noise. We should note that although we are adding more information to the system through the noise modulation, it is not obvious that it will enhance the additive SR effect. For instance, it was shown in Refs. 14, 15 that there is no SR effect with noise modulation in a symmetric double-well system.

We first consider signal-modulated noise. Consider a modulated noise intensity $D' = D/(1-\delta S(t))$, where $S(t)$ is presumed to have zero mean. The noise intensity increases (decreases) when the signal increases (decreases). For small signal amplitudes (or small δ), this is equivalent to a linear modulation $D(1+\delta S(t))$. We should note that the FHN equation (3.1) using D' now has a noise term $\xi(t)$ with a constant amplitude contribution and an amplitude-modulated contribution. Placing this into Eq. (3.3) yields the new rate expression

$$\langle R(t)\rangle \simeq Q\ \exp[-\theta/D + \Delta'/DS(t)], \tag{3.9}$$

where $\Delta' = \Delta + \delta\theta$ and terms of order $(\delta S(t))^2$ have been ignored. Substituting Δ' for Δ into Eqs. (3.7) and (3.8) gives the dependence of max(C_0) and max(C_1) on δ:

$$\max(C_0) \simeq Q(\Delta+\theta\delta)\theta^{-1}\ \exp(-1)\overline{S^2(t)}, \tag{3.10}$$

$$\max(C_1) \simeq \left[1 + \frac{K_1\theta^2\exp(2)}{4(\Delta+\theta\delta)^2 Q\overline{S^2(t)}}\right]^{-1/2}. \tag{3.11}$$

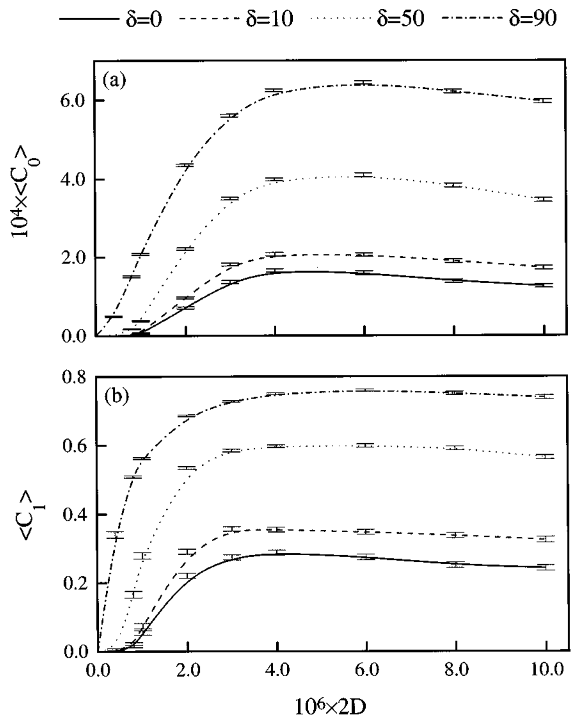

FIG. 1. Coherence measures (a) $C_0(D)$ and (b) $C_1(D)$ for various δ values for the FHN neuronal model with signal-modulated noise. The parameter values used in the simulations were $\epsilon=0.005$, $A_T=-5/(12\sqrt{3})$, and $B=0.07$ (Refs. 5, 8). The input signal $s(t)$ was Gaussian noise with a correlation time of 20 s followed by a 10 s Hanning filter. The same input signal $S(t)$, with variance $\sigma_S^2=1.5\times10^{-5}$ and total time length = 300 s, was used for all results presented. (All error bars indicate the standard error.)

We see that max(C_0) increases linearly with δ, and max(C_1) increases linearly for small δ but then saturates to unity. [It should be noted that the theoretical maxima are likely not to be attained because the expressions break down for $1-\delta S(t)<0$.] For larger δ, terms quadratic in $\delta S(t)$ will become more important so we should see deviations from the behavior given by Eqs. (3.10) and (3.11).

To test these predictions, we performed numerical simulations on the FitzHugh-Nagumo (FHN) neuronal model[22] for various δ values.[23] Plots of C_0 and C_1 for various values of δ are given in Fig. 1. As δ increases, both measures show clear increases, with C_1 attaining a value close to 0.8 for $\delta=90$. [For larger values of δ, the condition $1-\delta S(t)<1$ is violated.] In Fig. 2, max(C_0) and max(C_1) are plotted versus δ. We see that C_0 increases linearly, with a slight deviation for larger δ values; it can also be seen that C_1 increases linearly and then saturates, as expected. The solid lines are theoretical fits from Eqs. (3.10) and (3.11). No free parameters were used to fit the curves.

We next consider enhancement by modulating the noise intensity with the firing rate itself. Consider the noise intensity with the form $D' = D/(1-\delta Dr(t))$, where $r(t) = (R(t)-\langle\overline{R(t)}\rangle)/\langle\overline{R(t)}\rangle$. This expression can be substituted into Eqs. (2.1) and (2.2) and then maximized with respect to D. The equations need to be solved self-consistently. However, for small signal amplitudes, the rate expression, Eq. (3.3), can be expanded in $S(t)/D$, yielding $r(t)\simeq\Delta S(t)/D$. This then results in a modulated noise intensity of approxi-

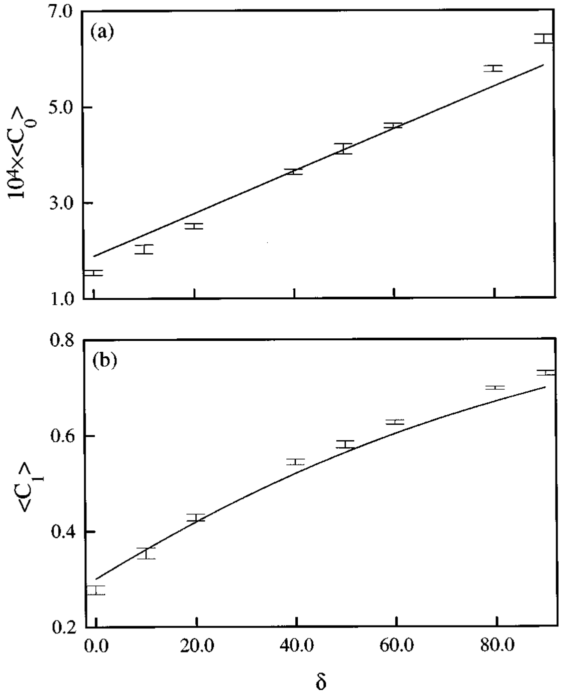

FIG. 2. Maximum coherence (a) $\max(C_0)$ and (b) $\max(C_1)$ versus δ for the FHN neuronal model with signal-modulated noise. The solid lines are the theoretical fits from Eqs. (3.8) and (3.9), where the simulation parameters listed in Fig. 1 and Refs. 5, 8 give $\Delta=1.27\times10^{-4}$, $\Theta=2.97\times10^{-6}$, $Q=0.80$, and $K_1=0.12$.

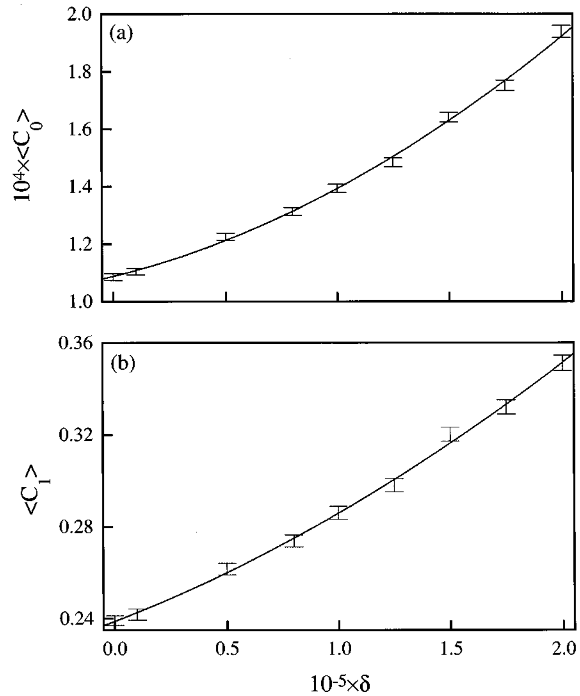

FIG. 3. Maximum coherence (a) $\max(C_0)$ and (b) $\max(C_1)$ versus δ for the FHN neuronal model with rate-modulated noise. In this simulation, the barrier height B (see Ref. 5) was 0.068 and $\overline{S^2}=9.4\times10^{-6}$.

mately $D'\simeq D/(1-\delta\Delta S(t))$, at lowest order. The previous analysis for signal-modulated noise then follows, culminating in the expressions given by Eqs. (3.7) and (3.8). The enhancement effect for rate-modulated noise, however, should not be as large as that for signal-modulated noise. This is mostly due to the fact that $r(t)$ will possess large fluctuations which occasionally violate the condition that $1-\delta Dr(t)>0$. Thus, the maximum value of δ that can be used is limited. These limitations could possibly be circumvented with a different form of rate modulation. We chose the above form because it corresponded to our analysis with signal-modulated noise. The precise way in which the rate modulates the noise intensity is not important — SR enhancement should occur provided the noise intensity is modulated, on average, in proportion to the input or output feedback.

Numerical simulations were performed on the FHN neuronal model with rate feedback. The spike train output from the FHN neuronal model was lowpass-filtered with a causal filter to obtain the time-varying mean firing rate signal.[24] This was then used to modulate the noise intensity. The theoretical rate from Eq. (3.3)[5,8] was used to zero-mean the rate. The dependence of $\max(C_0)$ and $\max(C_1)$ on δ are shown in Fig. 3. An enhancement is clearly observed, although it is not as dramatic as the enhancement for signal-modulated noise shown in Fig. 2. Deviations from linearity are likely due to the higher-order corrections not considered.

The results do not change significantly if there are time delays in the noise modulation, provided the delays are smaller than the correlation time of the input signal. Consider the situation where the noise is modulated by $D'=D/(1-\delta S(t-\tau))$, where τ is a time delay. The modified rate expression then becomes

$$\langle R\rangle\simeq Q\,\exp[-\theta/D+\Delta/DS(t)+\delta\theta/DS(t-\tau)],\quad(3.12)$$

which leads to

$$\overline{\langle R(t)\rangle S(t)}\simeq Q\,\exp(-\theta/D)[\Delta/D\overline{S^2(t)}+\delta\theta/D\overline{S(t)S(t-\tau)}].\quad(3.13)$$

We note that $\overline{S(t)S(t-\tau)}=\alpha\overline{S^2(t)}$, where α is a constant less than unity. Taking this into account, the expressions for $\max(C_0)$ and $\max(C_1)$ become

$$\max(C_0)\simeq Q\,\exp[-1]\overline{S^2(t)}(\Delta+\theta\alpha\delta)/\theta,\quad(3.14)$$

$$\max(C_1)\simeq\left[1+\frac{\delta^2\theta^2(1-\alpha^2)}{(\Delta+\theta\alpha\delta)^2}+\frac{K_1\theta^2\,\exp(2)}{4Q\overline{S^2(t)}(\Delta+\theta\alpha\delta)^2}\right]^{-1/2}.\quad(3.15)$$

These expressions show that the maximum values for C_0 and C_1 will still increase with δ provided $\alpha>0$. The maximum increase is attained with $\alpha=1$, which corresponds to no delay.

IV. CONCLUSIONS AND IMPLICATIONS

In this study, we showed that the conventional SR effect for aperiodic signals in a model neuron can be enhanced by modulating the input noise intensity with either the input signal or the unit's output rate signal. As noted earlier, this effect may be biologically relevant, i.e., it may be exploited in certain neurophysiological sensory systems.[17] Likewise, this effect could be incorporated into the design of signal-detection devices and techniques. For instance, it has been suggested that SR-based bioengineering techniques could be used to improve the function of neurophysiological sensory systems, such as the somatosensory system.[5,7,8] With such techniques, noise could be introduced artificially into sensory neurons in order to improve their abilities to detect arbitrary weak signals. The results of the present study indicate that the functional response of a given sensory neuron could be enhanced by modulating the intensity of the neuron's input noise with either the signal to be detected or the neuron's firing rate. This could be accomplished by using bioengineering sensors, e.g., to measure the neuron's output rate signal. As noted above, the time delays intrinsic to real-world feedback systems would not adversely affect the enhancement effect provided the delays were less than the correlation time of the input signal.

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